33rd INTERNATIONAL PHYSICS OLYMPIAD

BALI - INDONESIA

THEORETICAL COMPETITION

Tuesday, July 23rd, 2002

Please read this first:

1. The time available is 5 hours for the theoretical competition.
2. Use only the pen provided.
3. Use only the front side of the paper.
4. Begin each part of the problem on a separate sheet.
5. For each question, in addition to the ***answer sheets*** where you will write your solutions, there is some scrap paper for rough work.
6. Numerical results should be written with as many digits as are appropriate for the given data.
7. Write on the ***answer sheets*** whatever you consider is required for the solution of the question. Please use *as little text as possible*; express yourself primarily in equations, numbers, figures, and plots, and use the symbols that are given in the text to express physical quantities.
8. Fill in the boxes at the top of each sheet of paper used by writing your ***Country***, your student number (***Student No***.), the number of the question (***Question No***.), the progressive number of each sheet (***Page No***.), and the total number of blank sheets used for each question (***Total Pages***). Write the question number and the section letter of the part you are answering at the top of each sheet. If you use some blank sheets of paper for notes that you do not wish to be marked, put a large X across the entire sheet and do not include it in your numbering.
9. At the end of the exam, arrange all sheets for each problem *in the following order*:
    - Used Answer Sheets in order
    - Scrap papers
    - The sheets you do not wish to be marked
    - Unused sheets and the printed question

   Place the papers inside the envelope and leave everything on your desk. You are not allowed to take *any* sheets of paper out of the room.

I. Ground-Penetrating Radar

Ground-penetrating radar (GPR) is used to detect and locate underground objects near the surface by means of transmitting electromagnetic waves into the ground and receiving the waves reflected from those objects. The antenna and the detector are directly on the ground and they are located at the same point.

A linearly polarized electromagnetic plane wave of angular frequency ω propagating in the z direction is represented by the following expression for its field:

$$E = E_0 \, e^{-\alpha z} cos(\omega t - \beta z) \,, \qquad (1)$$

where E_o is constant, α is the attenuation coefficient and β is the wave number expressed respectively as follows

$$\alpha = \omega \left\{ \frac{\mu\varepsilon}{2} \left[\left(1 + \frac{\sigma^2}{\varepsilon^2 \omega^2} \right)^{1/2} - 1 \right] \right\}^{1/2} , \quad \beta = \omega \left\{ \frac{\mu\varepsilon}{2} \left[\left(1 + \frac{\sigma^2}{\varepsilon^2 \omega^2} \right)^{1/2} + 1 \right] \right\}^{1/2} \qquad (2)$$

with μ, ε, and σ denoting the magnetic permeability, the electrical permittivity, and the electrical conductivity respectively.

The signal becomes undetected when the amplitude of the radar signal arriving at the object drops below 1/e (≈ 37%) of its initial value. An electromagnetic wave of variable frequency (10 MHz – 1000 MHz) is usually used to allow adjustment of range and resolution of detection.

The performance of GPR depends on its resolution. The resolution is given by the minimum separation between the two adjacent reflectors to be detected. The minimum separation should give rise to a minimum phase difference of 180^o between the two reflected waves at the detector.

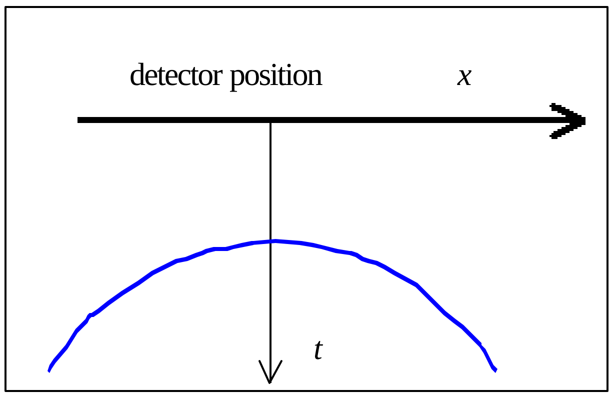
Questions:
(Given : $\mu_0 = 4\pi \text{ x}10^{-7}$ H/m and $\varepsilon_0 = 8.85 \text{ x}10^{-12}$ F/m)

1. Assume that the ground is non-magnetic ($\mu = \mu_0$) satisfying the condition $\left(\frac{\sigma}{\omega\varepsilon}\right)^2 \langle\langle 1$. Derive the expression of propagation speed v in terms of μ and ε, using equations (1) and (2) **[*1.0 pts*]**.

2. Determine the maximum depth of detection of an object in the ground with conductivity of 1.0 mS/m and permittivity of $9\varepsilon_0$, satisfying the condition $\left(\frac{\sigma}{\omega\varepsilon}\right)^2 \langle\langle 1$, (S=ohm^{-1} ; use $\mu=\mu_0$). **[*2.0 pts*]**

3. Consider two parallel conducting rods buried horizontally in the ground. The rods are 4 meter deep. The ground is known to have conductivity of 1.0 mS/m and permittivity of $9\varepsilon_0$. Suppose the GPR measurement is carried out at a position aproximately above one of the rod. Assume point detector is used. Determine the minimum frequency required to get a lateral resolution of 50 cm **[*3.5 pts*]**.

4. To determine the depth of a buried rod *d* in the same ground, consider the measurements carried out along a line perpendicular to the rod. The result is described by the following figure:



Graph of traveltime *t* vs detector position *x*, t_{min} = 100 ns.

Derive *t* as a function of *x* and determine *d* **[*3.5 pts*]**.

II. Sensing Electrical Signals

Some seawater animals have the ability to detect other creatures at some distance away due to electric currents produced by the creatures during the breathing processes or other processes involving muscular contraction. Some predators use this electrical signal to locate their preys, even when buried under the sands.

The physical mechanism underlying the current generation at the prey and its detection by the predator can be modeled as described by Figure II-1. The current generated by the prey flows between two spheres with positive and negative potential in the prey's body. The distance between the centers of the two spheres is l_s, each having a radius of r_s, which is much smaller than l_s. The seawater resistivity is ρ. Assume that the resistivity of the prey's body is the same as that of the surrounding seawater, implying that the boundary surrounding the prey in the figure can be ignored.

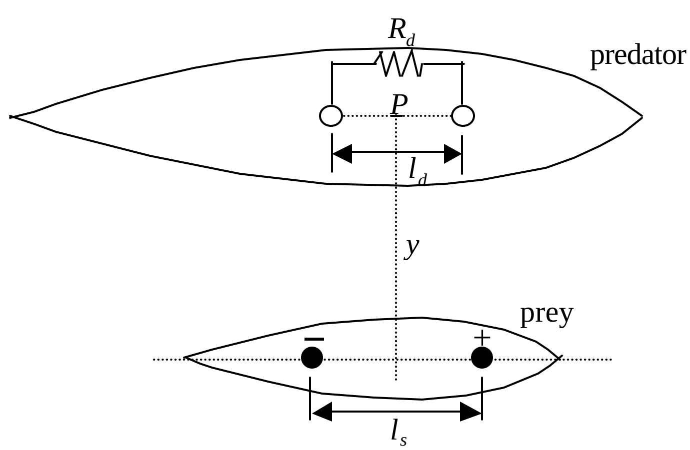


Figure II-1. A model describing the detection of electric power coming from a prey by its predator.

In order to describe the detection of electric power by the predator coming from the prey, the detector is modeled similarly by two spheres on the predator's body and in contact with the surrounding seawater, lying parallel to the pair in the prey's body. They are separated by a distance of l_d, each having a radius of r_d which is much smaller than l_d. In this case, the center of the detector is located at a distance y right above the source and the line connecting the two spheres is parallel to the electric field as shown in Figure II-1. Both l_s and l_d are also much smaller than y. The electric field strength along the line connecting the two spheres is assumed to be constant. Therefore the detector forms a closed circuit system connecting the prey, the surrounding seawater and the predator as described in Figure II-2.

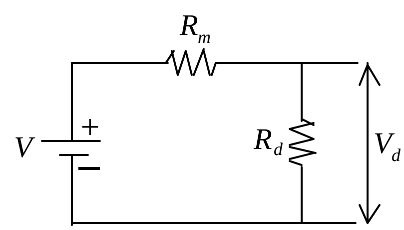


Figure II-2. The equivalent closed circuit system involving the sensing predator, the prey and the surrounding seawater.

In the figure, V is the voltage difference between the detector's spheres due to the electric field induced by the prey, R_m is the inner resistance due to the surrounding sea water. Further, V_d and R_d are respectively the voltage difference between the detecting spheres and the resistance of the detecting element within the predator.

Questions:

1. Determine the current density vector $\vec{j}$ (current per unit area) caused by a point current source I_s at a distance r in an infinite medium. **[*1.5 pts*]**

2. Based on the law $\vec{E} = r\vec{j}$, determine the electric field strength $\vec{E}_p$ at the middle of the detecting spheres (at point P) for a given current I_s that flows between two spheres in the prey's body **[*2.0 pts*]**.

3. Determine for the same current I_s, the voltage difference between the source spheres (V_s) in the prey **[*1.5 pts*]**. Determine the resistance between the two source spheres (R_s) **[*0.5 pts*]** and the power produced by the source (P_s) **[*0.5 pts*]**.

4. Determine R_m **[*0.5 pts*], V_d [*1.0 pts*]** in Figure II-2 and calculate also the power transferred from the source to the detector (P_d) **[*0.5 pts*]**.

5. Determine the optimum value of R_d leading to maximum detected power **[*1.5 pts*]** and determine also the maximum power **[*0.5 pts*]**.

III. A Heavy Vehicle Moving on An Inclined Road

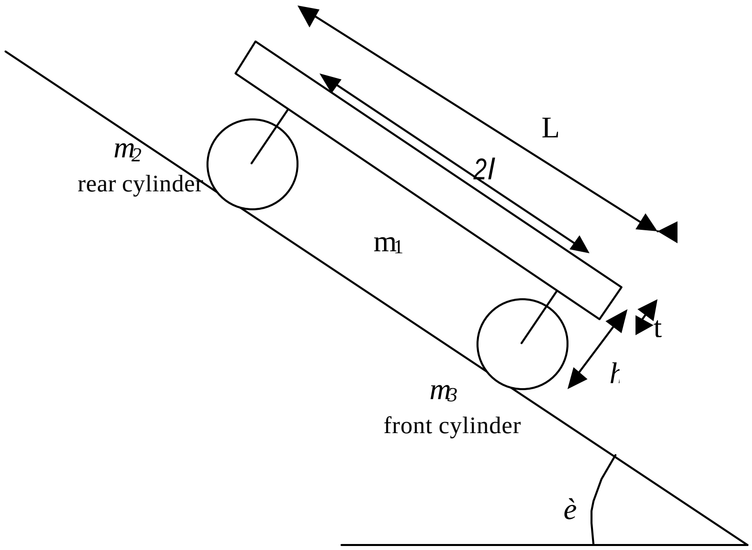


Figure III-1: A simplified model of a heavy vehicle moving on an inclined road.

The above figure is a simplified model of a heavy vehicle (road roller) with one rear and one front cylinder as its wheels on an inclined road with inclination angle of *è* as shown in Figure III-1. Each of the two cylinders has a total mass M(m_2=m_3=M) and consists of a cylindrical shell of outer radius R_o , inner radius R_i = 0.8 R_o and eight number of spokes with total mass 0.2 M. The mass of the undercarriage supporting the vehicle's body is negligible. The cylinder can be modeled as shown in Figure III-2. The vehicle is moving down the road under the influence of gravitational and frictional forces. The front and rear cylinder are positioned symmetrically with respect to the vehicle.

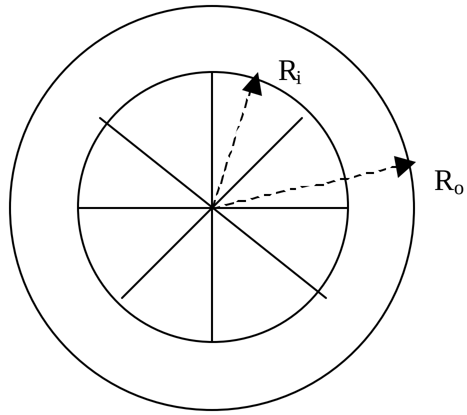


Figure III-2: A simplified model of the cylinders.

The static and kinetic friction coefficients between the cylinder *and* the road are m_s and m_k respectively. The body of the vehicle has a mass of 5M , length of L and thickness of t . The distance between the front and the rear cylinder is $2l$ while the distance from the center of cylinder to the base of the vehicle's body is h. Assume that the rolling friction between the cylinder and its axis is negligible.

Questions:

1. Calculate the moment of inertia of either cylinder **[*1.5 pts*]**.

2. Draw all forces that act on the body, the front cylinder, and the rear one. Write down equations of motion for each part of them **[*2.5 pts*]**.

3. The vehicle is assummed to move from rest, then freely move under gravitational influence. State all the possible types of motion of the system and derive their accelerations in terms of the given physical quantities **[*4.0 pts*]**.

4. Assume that after the vehicle travels a distance d by pure rolling from rest the vehicle enters a section of the road with all the friction coefficients drop to smaller constant values m_s' and m_k' such that the two cylinders start to slide. Calculate the linear and angular velocities of each cylinder after the vehicle has traveled a total distance of s meters. Here we assume that d and s is much larger than the dimension of vehicle **[*2.0 pts*]**