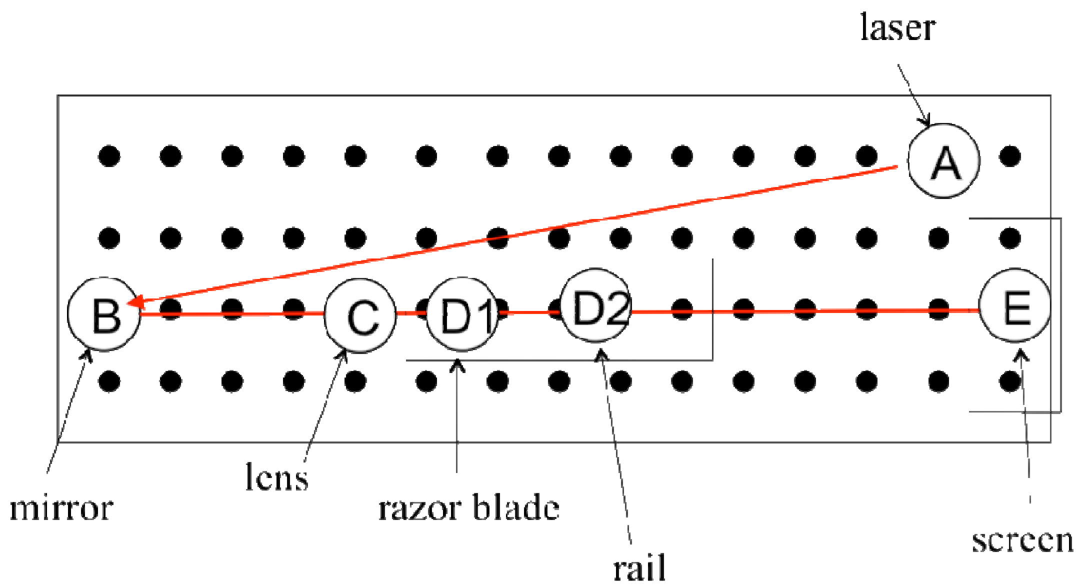
Answer Form

Experimental Problem No. 1

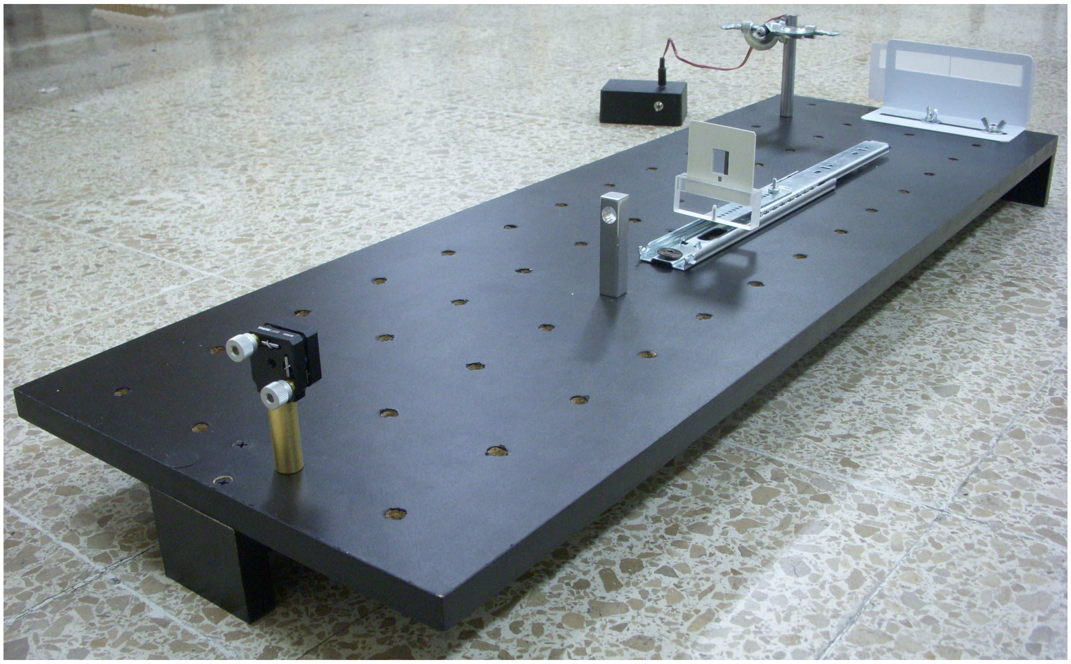
Diode laser wavelength

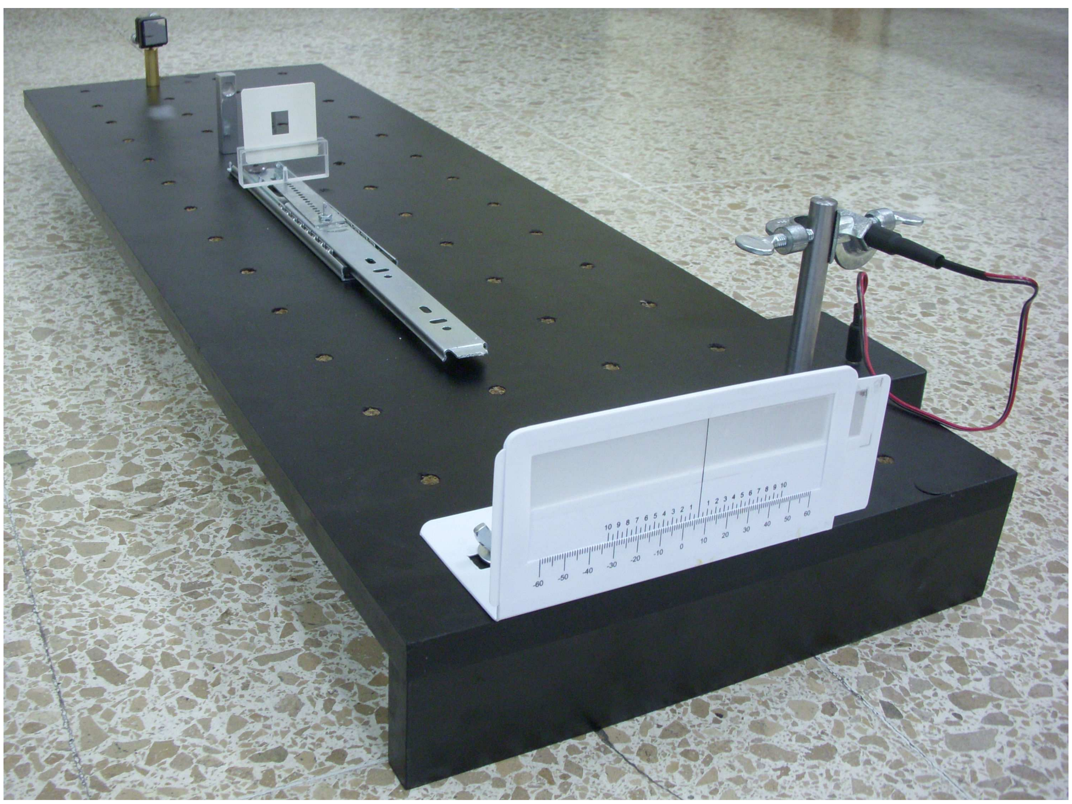
Task 1.1 Experimental setup.



(0.75)

1.1	Sketch the laser path in drawing of Task 1.1 and Write down the height h of the beam as measured from the table $h \pm \Delta h = (5.0 \pm 0.05) \times 10^{-2}$ m (0.25)	1.0





Experimental setup for measurement of diode laser wavelength

Task 1.2 Expressions for optical path differences.

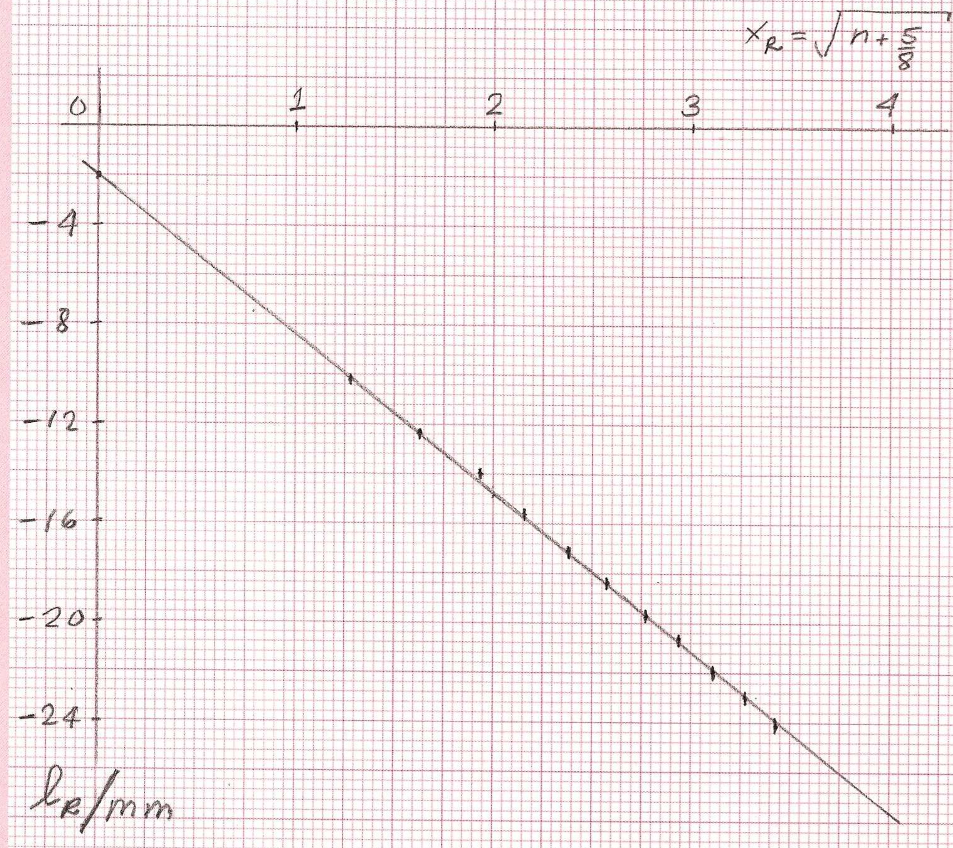
1.2	The path differences are Case I: (0.25) $\Delta_{\mathrm{I}}(n) = (BF + FP) - BP = (L_b - L_0) + \sqrt{L_0^2 + L_R^2(n)} - \sqrt{L_b^2 + L_R^2(n)}$ $= (L_b - L_0) + L_0\sqrt{1 + \frac{L_R^2(n)}{L_0^2}} - L_b\sqrt{1 + \frac{L_R^2(n)}{L_b^2}}$ using $\sqrt{1+x} \approx 1 + \frac{1}{2}x$ $\approx (L_b - L_0) + L_0\left(1 + \frac{1}{2}\frac{L_R^2(n)}{L_0^2}\right) - L_b\left(1 + \frac{1}{2}\frac{L_R^2(n)}{L_b^2}\right)$ $\Rightarrow \quad \Delta_{\mathrm{I}}(n) \approx \frac{1}{2}L_R^2(n)\left(\frac{1}{L_0} - \frac{1}{L_b}\right)$ Case II: (0.25) $\Delta_{\mathrm{II}}(n) = (FB + BP) - FP = (L_0 - L_a) + \sqrt{L_a^2 + L_L^2(n)} - \sqrt{L_0^2 + L_L^2(n)}$ $\approx (L_0 - L_a) + L_a\sqrt{1 + \frac{L_L^2(n)}{L_a^2}} - L_0\sqrt{1 + \frac{L_L^2(n)}{L_0^2}}$ using $\sqrt{1+x} \approx 1 + \frac{1}{2}x$ $\approx (L_0 - L_a) + L_a\left(1 + \frac{1}{2}\frac{L_L^2(n)}{L_a^2}\right) - L_0\left(1 + \frac{1}{2}\frac{L_L^2(n)}{L_0^2}\right)$ $\Rightarrow \quad \Delta_{\mathrm{II}}(n) \approx \frac{1}{2}L_L^2(n)\left(\frac{1}{L_a} - \frac{1}{L_0}\right)$	0.5

Task 1.3 Measuring the dark fringe positions and locations of the blade. Use additional sheets if necessary.

TABLE I

n	$(l_R(n) \pm 0.1) \times 10^{-3}$ m	$(l_L(n) \pm 0.1) \times 10^{-3}$ m	x_R	x_L
0	-7.5	1.1	0.791	0.935
1	-10.1	3.7	1.275	1.369
2	-12.4	6.4	1.620	1.696
3	-14.0	8.2	1.903	1.968
4	-15.6	10.0	2.151	2.208
5	-17.2	11.4	2.372	2.424
6	-18.4	12.2	2.574	2.622
7	-19.7		2.761	
8	-20.7		2.937	
9	-22.0		3.102	
10	-23.0		3.260	
11	-24.1		3.410	

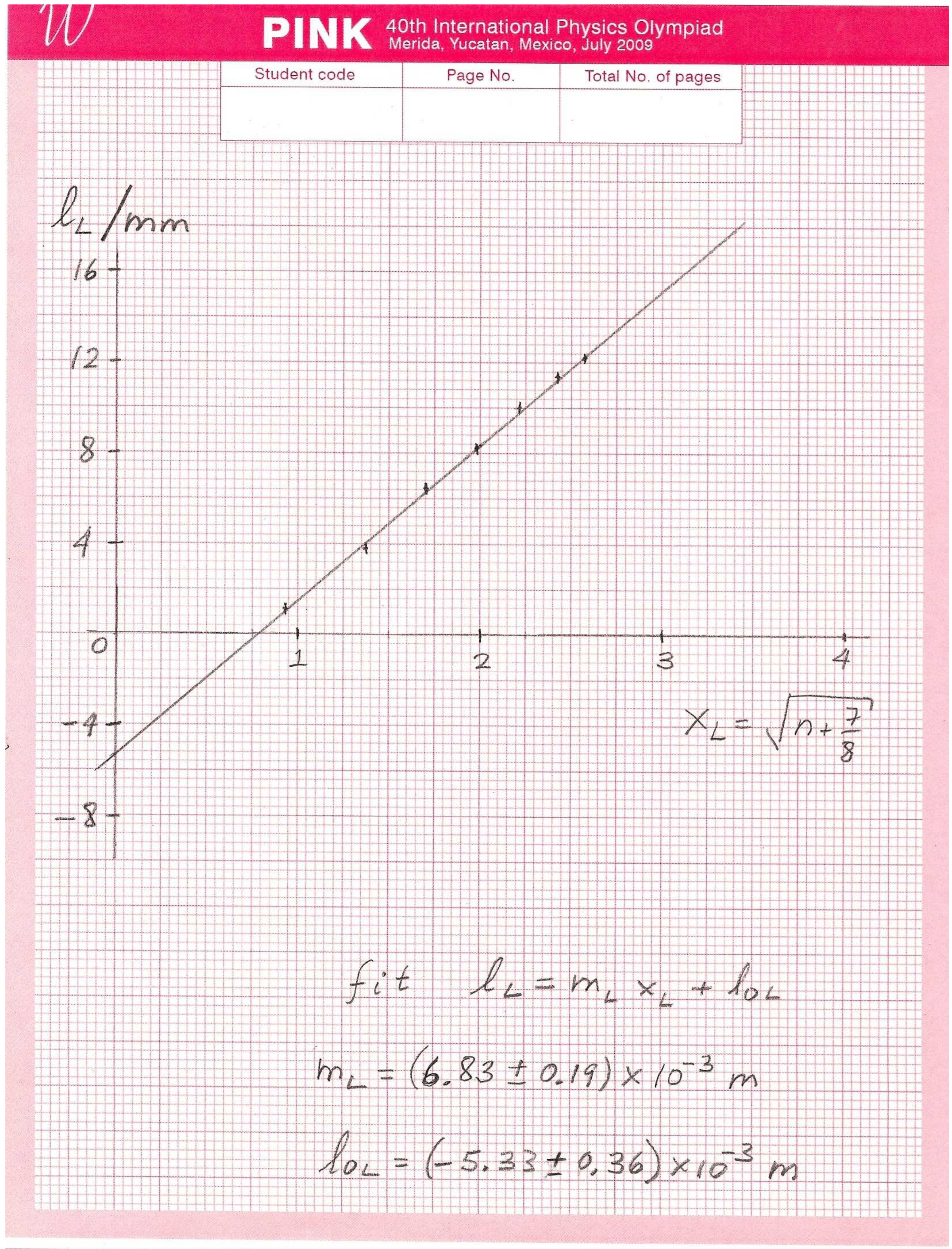
1.3	Report positions of the blade and their difference with higher precision: $L_b \pm \Delta L_b = (653 \pm 1) \times 10^{-3}$ m (0.25) LABEL (I) (measuring tape) $L_a \pm \Delta L_a = (628 \pm 1) \times 10^{-3}$ m (0.25) LABEL (I) (measuring tape) $d = L_b - L_a = (24.6 \pm 0.1) \times 10^{-3}$ m (0.25) LABEL (H) (caliper)	3.25



fit $l_R = m_R x_R + l_{OR}$

$m_R = (-6.39 \pm 0.07) \times 10^{-3}\ m$

$l_{OR} = (-2.06 \pm 0.17) \times 10^{-3}\ m$



Task 1.4 Performing a statistical and graphical analysis.

1.4	A procedure: From the condition of dark fringes and Task 1.2, we have $\frac{1}{2}L_R^2(n)\left(\frac{1}{L_0}-\frac{1}{L_b}\right)=\left(n+\frac{5}{8}\right)\lambda$ and $\frac{1}{2}L_L^2(n)\left(\frac{1}{L_a}-\frac{1}{L_0}\right)=\left(n+\frac{7}{8}\right)\lambda$ Using (1.5), $L_R(n)=l_R(n)-l_{0R}$ and $L_L(n)=l_L(n)-l_{0L}$ we can rewrite $\frac{1}{2}(l_R(n)-l_{0R})^2\left(\frac{1}{L_0}-\frac{1}{L_b}\right)=\left(n+\frac{5}{8}\right)\lambda$ $\Rightarrow \quad l_R(n)=\sqrt{\frac{2L_bL_0}{L_b-L_0}\lambda}\sqrt{n+\frac{5}{8}}+l_{0R}$ and $\frac{1}{2}(l_L(n)-l_{0L})^2\left(\frac{1}{L_a}-\frac{1}{L_0}\right)=\left(n+\frac{7}{8}\right)\lambda$ $\Rightarrow \quad l_L(n)=\sqrt{\frac{2L_aL_0}{L_0-L_a}\lambda}\sqrt{n+\frac{7}{8}}+l_{0L}$ These can be cast as equations of a straight line, $y=mx+b$. Case I: $y_R=l_R \qquad x_R=\sqrt{n+\frac{5}{8}} \qquad m_R=\sqrt{\frac{2L_bL_0}{L_b-L_0}\lambda} \qquad b_R=l_{0R}$ Case II: $y_L=l_L \qquad x_L=\sqrt{n+\frac{7}{8}} \qquad m_L=\sqrt{\frac{2L_aL_0}{L_0-L_a}\lambda} \qquad b_L=l_{0L}$ Perform least squares analysis of above equations. In Table I, we write down the values x_R and x_L. One finds: $m_R\pm\Delta m_R=(-6.39\pm0.07)\times10^{-3}$ m	3.25

$m_L \pm \Delta m_L = (6.83 \pm 0.19) \times 10^{-3}$ m

and (values of l_{0R} and l_{0L})

$l_{0R} \pm \Delta l_{0R} = b_R \pm \Delta b_R = (-2.06 \pm 0.17) \times 10^{-3}$ m

$l_{0L} \pm \Delta l_{0L} = b_L \pm \Delta b_L = (-5.33 \pm 0.36) \times 10^{-3}$ m

The equations used in the least squares analysis:

$$m = \frac{N\sum_{n=1}^{N} x_n y_n - \sum_{n=1}^{N} x_n \sum_{n'=1}^{N} y_{n'}}{\Delta}$$

$$b = \frac{\sum_{n=1}^{N} x_n^2 \sum_{n'=1}^{N} y_{n'} - \sum_{n=1}^{N} x_n \sum_{n'=1}^{N} x_{n'} y_{n'}}{\Delta}$$

where

$$\Delta = N\sum_{n=1}^{N} x_n^2 - \left(\sum_{n=1}^{N} x_n\right)^2$$

with N the number of data points.
The uncertainty is calculated as

$$(\Delta m)^2 = N\frac{\sigma^2}{\Delta} \quad , \quad (\Delta b)^2 = \frac{\sigma^2}{\Delta}\sum_{n=1}^{N} x_n^2 \qquad \text{with,}$$

$$\sigma^2 = \frac{1}{N-2}\sum_{n=1}^{N} (y_n - b - m x_n)^2$$

REFERENCE: P.R. Bevington, *Data Reduction and Error Analysis for the Physical Sciences*, McGraw-Hill, 1969.

Task 1.5 Calculating λ.

1.5	From any slope and the value of L_0 one finds, $\lambda = \frac{L_b - L_a}{2 L_a L_b} \frac{m_R^2 m_L^2}{m_R^2 + m_L^2}$ Using the suggestion to replace $d = L_b - L_a$, we can write	2.0

	$$\lambda = \frac{d}{2L_a L_b}\frac{m_R^2 m_L^2}{m_R^2 + m_L^2}$$ $$\lambda \pm \Delta\lambda = (663 \pm 25)\times 10^{-9}\ \text{m}$$ The uncertainty may range from 15 to 30 nanometers. A precise measurement of the wavelength is $\lambda \pm \Delta\lambda = (655 \pm 1)\times 10^{-9}$ m. The formula for the uncertainty, $$\Delta\lambda = \sqrt{\left(\frac{\partial\lambda}{\partial d}\right)^2 \Delta d^2 + \left(\frac{\partial\lambda}{\partial L_a}\right)^2 \Delta L_a^2 + \left(\frac{\partial\lambda}{\partial L_b}\right)^2 \Delta L_b^2 + \left(\frac{\partial\lambda}{\partial m_R}\right)^2 \Delta m_R^2 + \left(\frac{\partial\lambda}{\partial m_L}\right)^2 \Delta m_L^2}$$ one finds, $$\frac{\partial\lambda}{\partial d} = \frac{\lambda}{d}\ ,\ \frac{\partial\lambda}{\partial L_b} = \frac{\lambda}{L_b}\ ,\ \frac{\partial\lambda}{\partial L_a} = \frac{\lambda}{L_a}\quad \text{and}\quad \frac{\partial\lambda}{\partial m_R} = \frac{2m_L^2}{m_R}\frac{\lambda}{m_L^2 + m_R^2}$$ and analogously for the other slope. One can calculate directly these quantities. However, one may note that the errors due to L_a, L_b and d are negligible. Moreover, $m_R^2 \approx m_L^2$ and $L_a \approx L_b$. This implies, $\frac{\partial\lambda}{\partial m_R} \approx \frac{\lambda}{m_R} \approx \frac{\partial\lambda}{\partial m_L}$. Thus, $$\Delta\lambda \approx \sqrt{2}\frac{\lambda}{m_L}\Delta m_L \approx \left(25\times 10^{-9}\right)\ \text{m}$$	