General marking guidelines

1. Minor mistakes in the calculations e.g. copying expressions incorrectlyfrom line to line	Deduct 20% of the final answer
2. Missing units in the final numerical answers (for each part)	Deduct 0.1 point
3. Final answers (for each part) containing too few or too many significant figures (from +2 or -2 positions, say)	Deduct 0.1 point
4. Using wrong physical concepts (despite correct answers)	No points awarded
5. Error propagated from earlier parts: minor errors	Full points (except for the final answer of the same part. No marks are awarded for the final answer.)
6. Error propagated from earlier parts: major errors (such that the solution becomes trivial).	Deduct 20 - 50% for a particular part

Theoretical Question 1: A Three-body Problem and LISA

Questions	Points	Concepts/Details
1.1 (Total1.5)	**1.0**	**1.1a** Use the centripetal acceleration (0.5) and gravitational force (0.5) . (0.5 = 0.2 for concept + 0.3 for correct form)
	0.5	**1.1bAnswer:**Any of the three following answers: $\omega_0^{\ 2} = \frac{G(M+m)}{(R+r)^3}$, $\omega_0^{\ 2} = \frac{GM}{r(R+r)^2}$, $\omega_0^{\ 2} = \frac{Gm}{R(R+r)^2}$
1.2 (Total3.5)	**1.0**	**1.2a** Newton's 2[nd] law (0.2) for two components of radial forces (0.1 + 0.4 correct expression) and cirucular motion (0.1 + 0.2 correct expression) $\frac{GM\mu}{r_1^2}\cos\theta_1 + \frac{Gm\mu}{r_2^2}\cos\theta_2 = \mu\omega_0^{\ 2}\rho = \frac{G(M+m)\mu}{(R+r)^3}\rho$
	0.5	**1.2b**Newton's 1[st] law (0.1) for tangential forces (0.4 correct expression) $\frac{GM\mu}{r_1^2}\sin\theta_1 = \frac{Gm\mu}{r_2^2}\sin\theta_2$
	1.0	**1.2c** -Using at least two sine rules or sensible geometric relations (0.2)e.g. $\frac{\sin\psi_1}{\rho} = \frac{\sin\theta_1}{R}$ $\frac{\sin\psi_1}{r_2} = \frac{\sin(\theta_1+\theta_2)}{R+r}$ -showing adequate understanding of geometry and/or trigonometry in the problem (0.2) -algebraic manipulation to find a correct expression for r_1 (0.2) -algebraic manipulation to find a correct expression for r_2 (0.2) -realize that $r_1 = r_2$ (0.2)
	0.4	**1.2d**use the cosine rule or algebraic manipulation to find ρ (0.4)
	0.6	**1.2eAnswer**: $r_1 = R+r$ (0.2) , $r_2 = R+r$ (0.2) $\rho = \sqrt{r^2 + rR + R^2}$ (0.2)

Questions	Points	Concepts/Details
1.3 (Total 3.2)	**0.8**	**1.3a** Express energy in terms of potential (0.2) (0.1 for each term) and kinetic energy (0.2) (0.1 for each term) and the conservation of angular momentum (0.4) (correct form of angular momentum 0.2 and correct substitution 0.2) to obtain $E=-\frac{GM\mu}{r_1}-\frac{Gm\mu}{r_2}+\frac{1}{2}\mu(\dot{\rho}^2+\frac{\rho_0{}^4\omega_0{}^2}{\rho^2})$
	0.5	**1.3b** Use the conservation of energy as $\frac{dE}{dt}=0$
	0.4	**1.3c** Use the equilateral triangle: $\Re^2=\rho^2+R^2$, (0.2) $\rho_0=\Re_0\cos 30°$ or equivalent (0.2)
	0.4	**1.3d** Express $\frac{d\Re}{dt}=\frac{d\Re}{d\rho}\frac{d\rho}{dt}=\frac{\rho}{\Re}\frac{d\rho}{dt}$ (0.2)to obtain $\frac{d^2\rho}{dt^2}=-GM(\frac{\rho}{r_1^3}+\frac{\rho}{r_2^3})+2\frac{\rho_0{}^4\omega_0{}^2}{\rho^3}$ or $\frac{d^2\rho}{dt^2}=-\frac{2GM}{\Re^3}\rho+\frac{\rho_0{}^4\omega_0{}^2}{\rho^3}$ (0.2)
	0.3	**1.3e** Express perturbation for radial components $\Re$ and ρ $\rho=\rho_0(1+\frac{\Delta\rho}{\rho_0})$ (0.1), $\Delta\Re=\frac{\rho_0}{\Re_0}\Delta\rho$ (0.2)
	0.5	**1.3f** -Substitute $\Re$ and ρ to obtain expression for a simple harmonic motion $\frac{d^2\Delta\rho}{dt^2}\propto-\Delta\rho$ (0.3) - Express angular frequency of oscillation in terms of ω_0 only (0.2)
	0.3	**1.3g Answer:** frequency of oscillation $\frac{\sqrt{7}}{2}\omega_0$
1.3 (Total 3.2) Alternate Solution Marking Scheme	**0.4**	**1.3.2a** Initial Condition for $\omega_0=\frac{G(M+M)}{(R+R)^3}=\frac{GM}{4R^3}$ (0.2) $\rho_0=\sqrt{3}R$ (0.2)
	0.1	**1.3.2b** Evidence of perturbed radial distance e.g. $\rho=\sqrt{3}R+\zeta$ (0.1)
	0.3	**1.3.2c** Use of Newton's 2nd law
	0.6	**1.3.2d** Gravitational force(0.2)= mass(acceleration due to change in ρ (0.2) +centripetal force acceleration(0.2)). Note: For each 0.2 point 0.1 will be awarded for evidence for using correct concept and 0.1 for the correct expression)
	0.2	**1.3.2e** Correct distance between μ and M $\sqrt{R^2+(\sqrt{3}R+\zeta)^2}$
	0.2	**1.3.2f**Correctly project force into radial direction.

Questions	Points	Concepts/Details
	0.4	**1.3.1g**Using conservation of angular momentum (0.2) to obtain relationship between ω_0 and ω (0.2)
	0.7	**1.3.1h**Applying $\zeta^2 \approx 0$ approximation (0.1)and using binomial expansion(0.1) and algebraic manipulation (0.2) to obtain simple harmonic equation of ζ (0.3)
	0.3	**1.3.1i Answer** $\omega = \frac{\sqrt{7}}{2}\omega_0$
1.4 (Total1.8)	**0.4**	**1.4a** Find the angular velocity $\omega = \frac{2\pi}{T}$ using $T = 365 \times 24 \times 60 \times 60$ s (0.2 for correct relation between and T and ω and 0.2 for knowing numerical value of period = 1yr)
	0.5	**1.4b** Apply the circulation motion (0.1) and find the correct expression for radius (0.2) for each spacecraft to obtain $v = \omega \frac{L}{2\cos 30^\circ}$ (0.2)
	0.6	**1.4c** Correct expression of relative velocity e.g $\vec{v}_{BC} = \vec{v}_B - \vec{v}_C$ (0.1) using drawing or vectors for each $\vec{v}_B$ (0.2) and $\vec{v}_C$ (0.2) $\vec{v}_{BC} = -2v\sin 60^\circ \hat{\mathbf{j}} = -\sqrt{3}v\hat{\mathbf{j}}$ or $v_{BC} = \sqrt{3}v$ (0.1)
	0.3	**1.4d Answer:** $v_{BC} = 996$ m/s $\approx 1.0 \times 10^3$ m/s
1.4 note		Note 1.4a and 1.4b: Total of 0.9 will be awarded for any correct method for finding v from T. Points for the alternate solution using $v_{BC} = \omega L$ (axis of rotation is at one of the spacecrafts) will be given equivalently to the former solution.