Theoretical Question 2: An Electrified Soap Bubble

Questions	Points	Concepts/Details
2.1 **(Total 1.7)**	**0.3**	**2.1a** Know that the difference between pressure (or force) inside and outside the bubble comes from the surface pressure.
	0.3	**2.1b** Surface tension with **two** surfaces.
	0.5	**2.1c** use the concept of surface tension $dE = \gamma dA$ with correct $dA = d(4\pi r^2)$ (0.2) $dE = Fdr = \Delta PA dr$ (0.3) (other methods are also acceptable e.g. $F = \gamma L \frac{dE}{dx} = \gamma \frac{dA}{dx}$) If the sign of surface tension pressure is wrong, no mark awards.
	0.3	**2.1d** Correct usage of Ideal gas equation (0.1) $P = \frac{\rho RT}{M}$ (0.2 correct expression)
	0.3	**2.1e Answer:** $\frac{\rho_i T_i}{\rho_a T_a} = \left[1 + \frac{4\gamma}{R_0 P_a}\right]$ -If the sign of surface tension pressure is wrong, no mark awards. -No double penalty from part 2.1b - The term t cannot be included in this part since problem specify so
2.2 **(Total 0.4)**	**0.4**	**2.2a Answer:** $\frac{\rho_i T_i}{\rho_a T_a} - 1 = 0.0001$ For the answer ≥ 1: -0.2 major error 50% For the answer ≥ 0.5: -0.1 major error 25%
2.3 **(Total 2.0)**	**0.6**	**2.3a** Total weight from the mass of the bubble (0.2) and the inside air pulling downward (0.3), and substitute for ρ_i (0.1): $W = 4\pi R_0^2 \rho_s t g + \frac{4}{3}\pi R_0^3 \frac{\rho_a T_a}{T_i}\left[1 + \frac{4\gamma}{R_0 P_a}\right] g$ - In case that the student doesn't include the surface tension term, deduct 0.3 point if the answer in 2.2a is greater than 1. (a major error) Otherwise, full points.
	0.6	**2.3b** Use $B = \rho_a g V$ (0.3) Use the correct volume term (0.3) $\frac{4}{3}\pi R_0^3$. The term $R_0 + t$ instead of R_0 is acceptable
	0.4	**2.3c** Setting up $B = W$ or $B \geq W$.
	0.4	**2.3d Answer:** $T_i \geq 307.1$ K - The range of answer within [305,309] is acceptable.

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2.4 **(Total 1.6)**	**0.5**	**2.4a** Setting the force balance $F \geq W - B$ ("equal sign" also acceptable) (0.5, but only give 0.1 for incorrect sign).
	0.2	**2.4b** Correct expressions for the weight of the bubble (0.1) plus the inside air (0.1). $W = \left(4\pi R_0^2\ \rho_s t + \frac{4}{3}\pi R_0^3 \rho_i\right) g$
	0.5	**2.4c** Thermal equilibrium means $T_i = T_a$ (0.3) and substitute for ρ_i (0.2)
	0.4	**2.4d Answer**: $u \geq \frac{4R_0\ \rho_s t g}{6\eta} + \frac{\frac{4}{3} R_0^2 \rho_a g\left(\frac{4\gamma}{R_0 P_a}\right)}{6\eta}$ - If the term due to surface tension is neglected in 2.3a, the second term above can also be neglected - In 2.3a, if the student uses $R_0 + t$ instead of R_0, there will be an additional third term. That is acceptable.
2.5 **(Total 0.4)**	**0.4**	**2.5a Answer**: $u \geq 0.36$ m/s or $u_{\min} = 0.36 m/s$ -The numerical value in range of [0.35,0.37] is acceptable
2.6 **(Total 2.0)** **Method A**	**0.2**	**2.6a** Gaussian Law leading to the electric field outside the soap bubble: $E_q = \frac{\sigma}{\varepsilon_0}$ *If no factor 1/2 , no mark for the following part b,c
	0.2	**2.6b** Gaussian Law leading to the electric field on the pill box: $E_\sigma = \frac{\sigma}{2\varepsilon_0}$
	0.3	**2.6c** Symmetry lead to the electric field from all other parts of the film excluding the pill box itself: $E = \frac{1}{2\varepsilon_0}\frac{q}{4\pi R_1^2}$
Or **Method B**	**0.2**	**2.6a** Charge on a small stripe of the bubble film: $\delta q = \left(\frac{q}{4\pi R^2}\right) 2\pi R \sin\theta . R\delta\theta$
	0.2	**2.6b** Form the integration with a correct stripe.
	0.3	**2.6c** Do the integration correctly: $E = \frac{1}{2\varepsilon_0}\frac{q}{4\pi R_1^2}$
2.6 cont.	**0.3**	**2.6d** Repulsive force per unit area of the bubble: $\frac{\left(q/4\pi R_1^2\right)^2}{2\varepsilon_0}$

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	0.4	**2.6e** Use Boyle's Law to find the new pressure.
	0.3	**2.6f** Balancing the pressurized force pushing inward and outward
	0.3	**2.6g Answer**: $\left(\frac{R_1}{R_0}\right)^4 - \left(\frac{R_1}{R_0}\right) - \frac{q^2}{32\pi^2\varepsilon_0 R_0^4 P_a} = 0$
2.7 (Total 0.7)	0.3	**2.7a** Apply the approximation: $R_1 = R_0 + \Delta R,\ \Delta R << R_0$
	0.4	**2.7b Answer**: $R_1 \approx R_0\left(1 + \frac{q^2}{96\pi^2\varepsilon_0 R_0^4 P_a}\right)$
2.8 (Total 1.2)	0.7	**2.8a** Newton's Law (0.3). The balance between the weight (0.2) and the buoyancy (0.2). - Check the correct formula for weigh and buoyant force from (21) in the solution. No double penalty for the wrong formula of W from 2.4b. - If the student write down the weigh W in term of the new radius, R_1, and new density, that solution is acceptable too as long as it is correct.
	0.3	**2.8b Answer**: $q^2 \geq \frac{96\pi^2 R_0^3 \rho_s t \varepsilon_0 P_a}{\rho_a}$
	0.2	**2.8c Answer**: $q \approx 256\times10^{-9}$ C ≈ 256 nC -The numerical value in range of [250,260]nC is acceptable.