33rd
INTERNATIONAL
PHYSICS OLYMPIAD
2002
BALI - INDONESIA

THEORETICAL COMPETITION

Tuesday, July 23rd, 2002

Solution I: Ground-Penetrating Radar

1. Speed of radar signal in the material v_m:

$$\omega t - \beta z = \text{ constant } \rightarrow \beta z = \text{-constant } + \omega t \quad \text{(0.2 pts)}$$

$$v_m = \frac{\omega}{\beta}$$

$$v_m = \frac{1}{\omega\left\{\frac{\mu\varepsilon}{2}\left[(1+\frac{\sigma^2}{\varepsilon^2\omega^2})^{1/2}+1\right]\right\}^{1/2}} \quad \text{(0.4 pts)}$$

$$v_m = \frac{1}{\left\{\frac{\mu\varepsilon}{2}(1+1)\right\}^{1/2}} = \frac{1}{\sqrt{\mu\varepsilon}} \quad \text{(0.4 pts)}$$

2. The maximum depth of detection (skin depth, *d*) of an object in the ground is inversely proportional to the attenuation constant:

(0.5 pts) **(0.3 pts)** **(0.2 pts)**

$$d = \frac{1}{a} = \frac{1}{w\left\{\frac{me}{2}\left[\left(1+\frac{s^2}{e^2w^2}\right)^{1/2}-1\right]\right\}^{1/2}} = \frac{1}{w\left\{\frac{me}{2}\left[\left(1+\frac{1}{2}\frac{s^2}{e^2w^2}\right)-1\right]\right\}^{1/2}} = \frac{1}{w\left\{\frac{me}{2}.\frac{1}{2}\frac{s^2}{e^2w^2}\right\}^{1/2}}$$

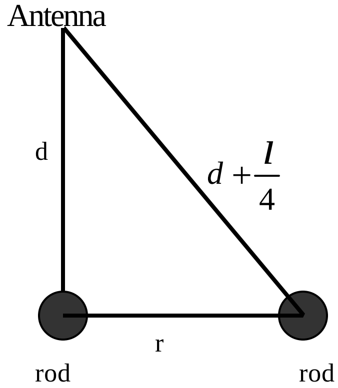
$$d = \left(\frac{2}{s}\right)\left(\frac{e}{m}\right)^{1/2}.$$

Numerically $d = \frac{\left(5.31\sqrt{e_r}\right)}{s}$ m, where s is in mS/m. **(0.5 pts)**

For a medium with conductivity of 1.0 mS/m and relative permittivity of 9, the skin depth

$$d = \frac{\left(5.31\sqrt{9}\right)}{1.0} = 15.93 \text{ m}$$ **(0.3 pts) + (0.2 pts)**

3. Lateral resolution:



$$r^2 + d^2 = \left(d + \frac{\lambda}{4}\right)^2$$

$$r = \left(\frac{\lambda d}{2} + \frac{\lambda^2}{16}\right)^{1/2}$$

(1.0 pts)

r =0.5 m, d =4 m: $\frac{1}{2} = \left(\frac{4\lambda}{2} + \frac{\lambda^2}{16}\right)^{1/2}$, $\lambda^2 + 32\lambda - 4 = 0$ **(0.5 pts)**

The wavelength is λ=0.125 m. **(0.3 pts) + (0.2 pts)**

The propagation speed of the signal in medium is

$$v_m = \frac{1}{\sqrt{\mu\varepsilon}} = \frac{1}{\sqrt{\mu_o \mu_r \varepsilon_o \varepsilon_r}} = \frac{1}{\sqrt{\mu_o \varepsilon_o}} \frac{1}{\sqrt{\mu_r \varepsilon_r}}$$

$$v_m = \frac{c}{\sqrt{\mu_r \varepsilon_r}} = \frac{0.3}{\sqrt{\varepsilon_r}} \text{ m/ns} \text{ , where } c = \frac{1}{\sqrt{\mu_o \varepsilon_o}} \text{ and } \mu_r = 1$$

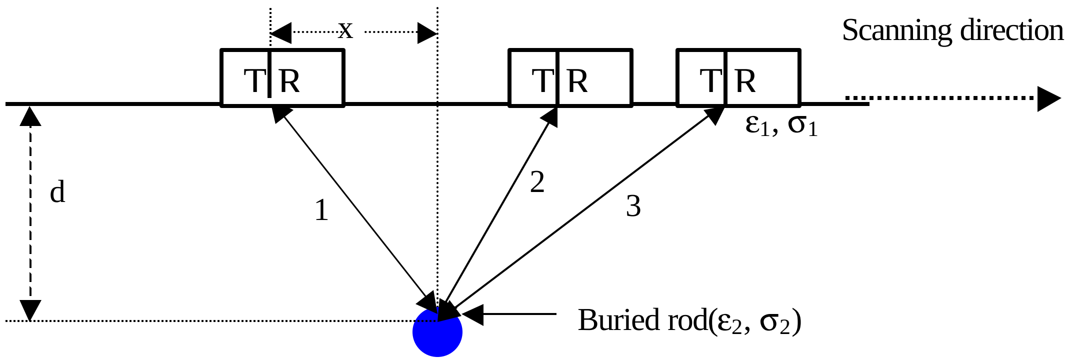
$$v_m = 0.1 \text{ m/ns} = 10^8 \text{ m/s}$$ **(0.5 pts)**

The minimum frequency need to distinguish the two rods as two separate objects is

$$f_{\min} = \frac{v}{\lambda}$$ **(0.5 pts)**

$$f_{\min} = \frac{\frac{0.3}{\sqrt{9}}}{0.125} x 10^9 \text{ Hz} = 800 \text{ MHz}$$ **(0.3 pts) + (0.20 pts)**

4. Path of EM waves for some positions on the ground surface

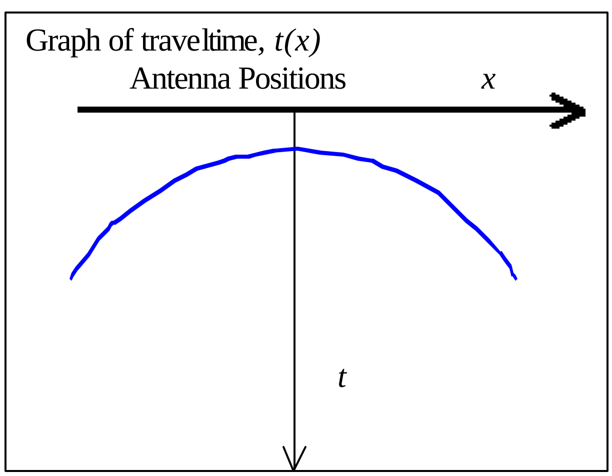


The traveltime as function of x is

$$\left(\frac{t\ v}{2}\right)^2 = d^2 + x^2 ,$$ **(1.0 pts)**

$$t(x) = \sqrt{\frac{4d^2 + 4x^2}{v}}$$ **(1.0 pts)**

$$t(x) = \frac{2\sqrt{e_{1r}}}{0.3}\sqrt{d^2 + x^2}$$



For x =0 **(1.0 pts)**

100 = 2×(3/0.3) d

d = 5 m **(0.5 pts)**

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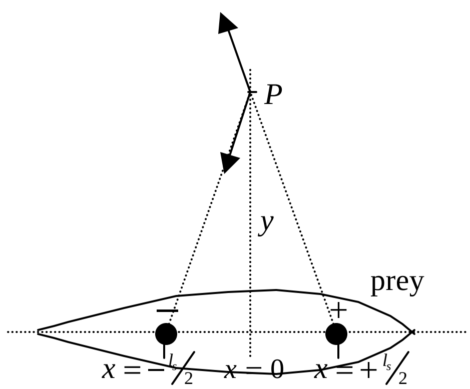
Solution II: Sensing Electrical Signals

1. When a point current source I_s is in infinite isotropic medium, the current density vector at a distance r from the point is

$$\vec{j} = \frac{I_s}{4pr^3}\vec{r}$$

[+1.5 pts] (without vector notation, -0.5 pts)

2.



Assuming that the resistivities of the prey body and that of the surrounding seawater are the same, implying the elimination of the boundary surrounding the prey, the two spheres seem to be in infinite isotropic medium with the resistivity of r. When a small sphere produces current at a rate I_s, the current flux density at a distance r from the sphere's center is also

$$\vec{j} = \frac{I_s}{4pr^3}\vec{r}$$

The seawater resistivity is r, therefore the field strength at r is

$$\vec{E}(\vec{r}) = r\vec{j} = \frac{rI_s}{4pr^3}\vec{r} \qquad \text{[+0.2 pts]}$$

In the model, we have two small spheres. One is at positive voltage relative to the other therefore current I_s flows from the positively charged sphere to the negatively charged sphere. They are separated by l_s. The field strength at P(0,y) is:

$$\vec{E}_p = \vec{E}_+ + \vec{E}_- \qquad \text{[+0.8 pts]}$$

$$= \frac{rI_s}{4p}\left[\frac{1}{\left(\left(\frac{l_s}{2}\right)^2 + y^2\right)^{\frac{3}{2}}}\left(-\frac{l_s}{2}i + yj\right) + \frac{1}{\left(\left(\frac{l_s}{2}\right)^2 + y^2\right)^{\frac{3}{2}}}\left(-\frac{l_s}{2}i - yj\right)\right]$$

$$= \frac{rI_s}{4p}\left[\frac{l_s(-i)}{\left(\left(\frac{l_s}{2}\right)^2 + y^2\right)^{\frac{3}{2}}}\right]$$

$$\vec{E}_p \approx \frac{rI_s l_s}{4py^3}(-i) \quad \text{for} \quad \text{ls} << \text{y} \qquad \text{[+1.0 pts]}$$

3. The field strength along the axis between the two source spheres is:

$$\vec{E}(x) = \frac{rI_s}{4p}\left(\frac{1}{\left(x - \frac{l_s}{2}\right)^2} + \frac{1}{\left(x + \frac{l_s}{2}\right)^2}\right)(-i) \qquad \text{[+0.5 pts]}$$

The voltage difference to produce the given current I_s is

$$V_s = \Delta V = V_+ - V_- = -\int_{\left(-\frac{l_s}{2}+r_s\right)}^{\left(\frac{l_s}{2}-r_s\right)} \vec{E}(x).d\vec{x} = -\frac{rI_s}{4p}\int\left(\frac{1}{\left(x - \frac{l_s}{2}\right)^2} + \frac{1}{\left(x + \frac{l_s}{2}\right)^2}\right)(-i).(idx) \quad \text{[+0.5 pts]}$$

$$= \frac{rI_s}{4p}\left[\frac{1}{-2+1}\left(\frac{1}{\left(\frac{l_s}{2} - r_s - \frac{l_s}{2}\right)} - \frac{1}{\left(-\frac{l_s}{2} + r_s - \frac{l_s}{2}\right)}\right) + \frac{1}{-2+1}\left(\frac{1}{\left(\frac{l_s}{2} - r_s + \frac{l_s}{2}\right)} - \frac{1}{\left(-\frac{l_s}{2} + r_s + \frac{l_s}{2}\right)}\right)\right]$$

$$= \frac{rI_s}{4p}\left(\frac{2}{r_s} - \frac{2}{l_s - r_s}\right) = \frac{2rI_s}{4p}\left(\frac{l_s - r_s - r_s}{(l_s - r_s)r_s}\right) = \frac{rI_s}{2pr_s}\left(\frac{l_s - 2r_s}{l_s - r_s}\right)$$

$$V_s = \Delta V \approx \frac{rI_s}{2pr_s} \quad \text{for} \quad l_s >> r_s. \qquad \text{[+0.5 pts]}$$

The resistance between the two source spheres is:

$$R_s = \frac{V_s}{I_s} = \frac{r}{2pr_s}$$

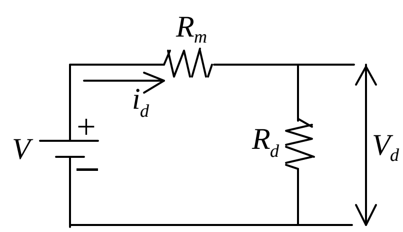
[+0.5 pts]

The power produced by the source is:

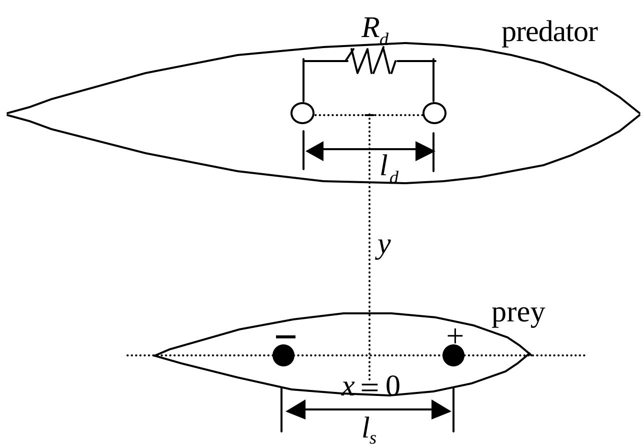
$$P = I_s V_s = \frac{rI_s^2}{2pr_s}$$

[+0.5 pts]

4.



V is the voltage difference between the detector's spheres due to the electric field induced by the prey, R_m is the inner resistance due to the surrounding sea water. V_d and R_d are respectively the voltage difference between the detecting spheres and the resistance of the detecting element within the predator and i_d is the current flowing in the closed circuit.



Analog to the resistance between the two source spheres, the resistance of the medium with resistivity r between the detector spheres, each having a radius of r_d is:

$$R_m = \frac{r}{2pr_d}$$

[+0.5 pts]

Since l_d is much smaller than y, the electric field strength between the detector spheres can be assumed to be constant, that is:

$$E = \frac{rI_s l_s}{4py^3}$$ [+0.2 pts]

Therefore, the voltage difference present in the medium between the detector spheres is:

$$V = El_d = \frac{rI_s l_s l_d}{4py^3}$$ [+0.3 pts]

The voltage difference across the detector spheres is:

$$V_d = V\frac{R_d}{R_d + R_m} = \frac{rI_s l_s l_d}{4py^3}\frac{R_d}{R_d + \dfrac{r}{2pr_d}}$$

[+0.5 pts]

The power transferred from the source to the detector is:

$$P_d = i_d V_d = \frac{V}{R_d + R_m}V_d = \left(\frac{rI_s l_s l_d}{4py^3}\right)^2 \frac{R_d}{\left(R_d + \dfrac{r}{2pr_d}\right)^2}$$

[+0.5 pts]

5. P_d is maximum when

$$R_t = \frac{R_d}{\left(R_d + \dfrac{r}{2pr_d}\right)^2} = \frac{R_d}{\left(R_d + R_m\right)^2}$$ is maximum [+0.5 pts]

Therefore,

$$\frac{dR_t}{dR_d} = \frac{1\left(R_d + R_m\right)^2 - R_d 2\left(R_d + R_m\right)}{\left(R_d + R_m\right)^4} = 0$$ [+0.5 pts]

$$\left(R_d + R_m\right) - 2R_d = 0$$

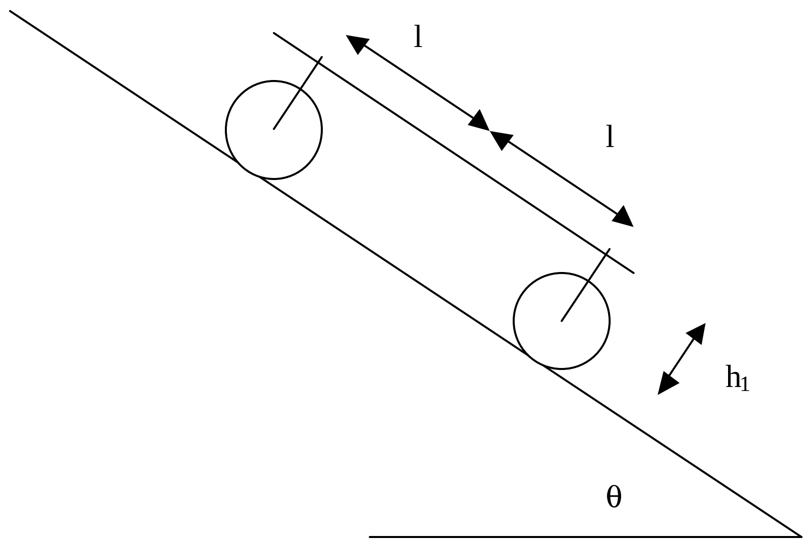
$$R_d^{optimum} = R_m = \frac{r}{2pr_d}$$ [+0.5 pts]

The maximum power is:

$$P_d^{\max imum} = \left(\frac{rI_s l_s l_d}{4py^3}\right)^2 \frac{pr_d}{2r} = \frac{r\left(I_s l_s l_d\right)^2 r_d}{32py^6}$$

[+0.5 pts]

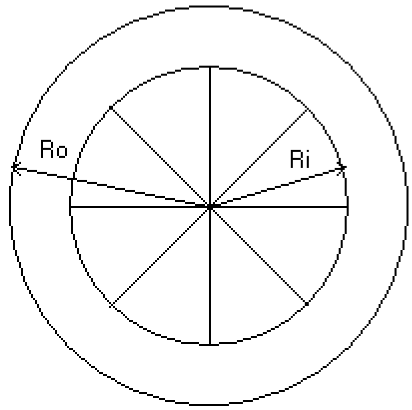
SOLUTION T3 : . A Heavy Vehicle Moving on An Inclined Road



To simplify the model we use the above figure with $h_1 = h+0.5\ t$
$R_o = R$

1. **Calculation of the moment inertia of the cylinder**

   R_i=0.8 R_o
   Mass of cylinder part : $m_{cylinder}$ =0.8 M
   Mass of each rod : m_{rod} = 0.025 M



$$I = \oint_{wholepart} r^2 dm = \oint_{cyl.shell} r^2 dm + \oint_{rod1} r^2 dm + ... + \oint_{rod\,n} r^2 dm \qquad 0.4 \text{ pts}$$

$$\oint_{cyl.shell} r^2 dm = 2\boldsymbol{ps} \int_{Ri}^{Ro} r^3 dr = 0.5\boldsymbol{ps}(R_o^4 - R_i^4) = 0.5 m_{cylinder}(R_o^2 + R_i^2)$$

$$= 0.5(0.8M)R^2(1 + 0.64) = 0.656MR^2 \qquad 0.5 \text{ pts}$$

$$\oint_{rod} r^2 dm = \boldsymbol{l} \int_0^{Rin} r^2 dr = \frac{1}{3}\boldsymbol{l}R_{in}^3 = \frac{1}{3}m_{rod}R_{in}^2 = \frac{1}{3}0.025M(0.64R^2) = 0.00533MR^2 \qquad 0.5 \text{ pts}$$

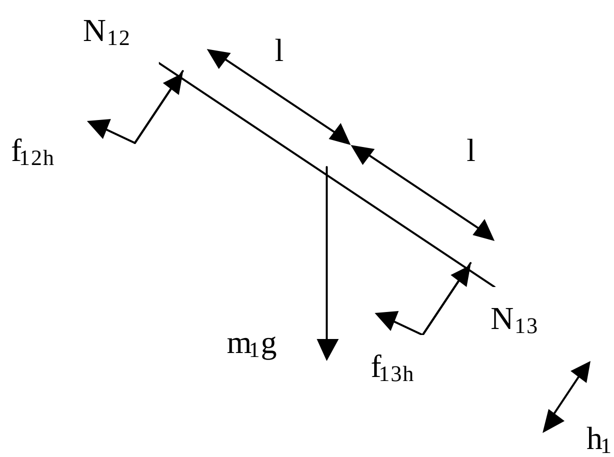
The moment inertia of each wheel becomes

$$I = 0.656MR^2 + 8x0.00533MR^2 = 0.7MR^2 \qquad 0.1 \text{ pts}$$

2. Force diagram and balance equations:

To simplify the analysis we devide the system into three parts: frame (part1) which mainly can be treated as flat homogeneous plate, rear cylinders (two cylinders are treated collectively as part 2 of the system), and front cylinders (two front cylinders are treated collectively as part 3 of the system).

Part 1 : Frame



0.4 pts

The balance equation related to the forces work to this parts are:

Required conditions:
Balance of force in the horizontal axis

$m_1 g \sin \theta - f_{12h} - f_{13h} = m_1 a$ (1) 0.2 pts

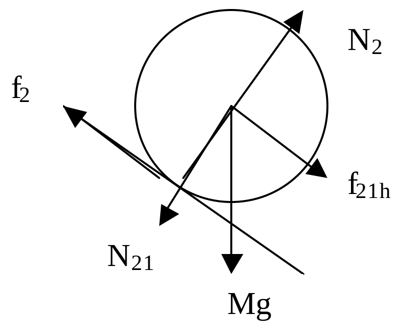
Balance of force in the vertical axis

$m_1 g \cos \theta = N_{12} + N_{13}$ (2) 0.2 pts

Then torsi on against O is zero, so that

$N_{12} l - N_{13} l + f_{12h} h_1 + f_{13h} h_1 = 0$ (3) 0.2 pts

Part two : Rear cylinder



0.25 pts

From balance condition in rear wheel :

$f_{21h} - f_2 + Mg \sin \theta = Ma$ (4) 0.15 pts

$N_2 - N_{21} - Mg \cos \theta = 0$ (5) 0.15 pts

For pure rolling:

$$f_2 R = I\alpha_2 = I \frac{a_2}{R}$$

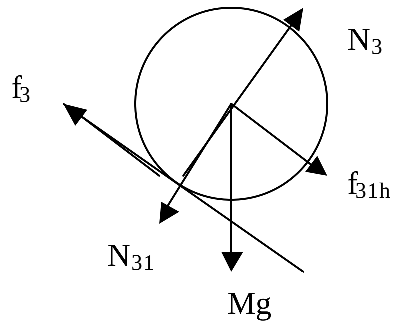
$$\text{or} \quad f_2 = \frac{I}{R^2} a \qquad (6)$$

For rolling with sliding:

$$F_2 = u_k N_2 \qquad (7)$$

0.2 pts

Part Three : Front Cylinder:



0.25 pts

From balance condition in the front wheel :

$f_{31h} - f_3 + Mg\sin\theta = Ma$ (8) 0.15 pts

$N_3 - N_{31} - Mg\cos\theta = 0$ (9) 0.15 pts

For pure rolling:

$$f_3 R = I\alpha_3 = I\frac{a_3}{R}$$

or $f_3 = \frac{I}{R^2}a$ (10)

For rolling with sliding:

$F_3 = u_k N_3$ (11)

0.2 pts

3. From equation (2), (5) and (9) we get

$m_1 g\cos\theta = N_2 - m_2 g\cos\theta + N_3 - m_3 g\cos\theta$

$N_2 + N_3 = (m_1+m_2+m_3)g\cos\theta = 7Mg\cos\theta$ (12)

And from equation (3), (5) and (8) we get

$(N_3 - Mg\cos\theta)l - (N_2 - Mg\cos\theta)l = h_1(f_2 + Ma - Mg\sin\theta + f_3 + Ma - Mg\sin\theta)$

$(N_3 - N_2) = h_1(f_2 + 2Ma - 2Mg\sin\theta + f_3)/l$

Equations 12 and 13 are given **0.25 pts**

CASE ALL CYLINDER IN PURE ROLLING

From equation (4) and (6) we get

$f_{21h} = (I/R^2)a + Ma - Mg\sin\theta$ (14) 0.2 pts

From equation (8) and (10) we get

$f_{31h} = (I/R^2)a + Ma - Mg\sin\theta$ (15) 0.2 pts

Then from eq. (1) , (14) and (15) we get

$5Mg\sin\theta - \{(I/R^2)a + Ma - Mg\sin\theta\} - \{(I/R^2)a + Ma - Mg\sin\theta\} = m_1 a$

$7\,Mg\sin\theta = (2I/R^2 + 7M)a$

$$a = \frac{7Mg\sin\theta}{7M + 2\dfrac{I}{R^2}} = \frac{7Mg\sin\theta}{7M + 2\dfrac{0.7MR^2}{R^2}} = 0.833g\sin\theta \quad (16) \quad 0.35 \text{ pts}$$

$$N_3 = \frac{7M}{2}g\cos\theta + \frac{h_1}{l}[(M + \frac{I}{R^2}) \times 0.833g\sin\theta - Mg\sin\theta]$$

$$= 3.5\text{Mg}\cos\theta + \frac{h_1}{l}[(M + 0.7M) \times 0.833g\sin\theta - Mg\sin\theta]$$

$$= 3.5\text{ Mg}\cos\theta + 0.41\frac{h_1}{l}Mg\sin\theta$$

$$N_2 = \frac{7M}{2}g\cos\theta - \frac{h_1}{l}[(\frac{I}{R^2} + M) \times 0.833g\sin\theta - Mg\sin\theta]$$

$$= 3.5\text{g}\cos\theta - \frac{h_1}{l}[(0.7M + M)\frac{7Mg\sin\theta}{0.7M + 7M} - 2Mg\sin\theta]$$

$$= 3.5\text{g}\cos\theta - 0.41\frac{h_1}{l}Mg\sin\theta$$

0.2 pts

The Conditions for pure rolling:

$f_2 \le \mu_s N_2$ and $f_3 \le \mu_s N_3$

$\frac{\mathrm{I}_2}{\mathrm{R}_2^2}\mathrm{a} \le \mu_s N_2$ and $\frac{\mathrm{I}_3}{\mathrm{R}_3^2}\mathrm{a} \le \mu_s N_3$ 0.2 pts

The left equation becomes

$$0.7M \times 0.833g\sin\theta \le \mu_s(3.5\text{Mg}\cos\theta - 0.41\frac{h_1}{l}Mg\sin\theta)$$

$$\tan\theta \le \frac{3.5\mu_s}{0.5831 + 0.41\mu_s\dfrac{h_1}{l}}$$

While the right equation becomes

$$0.7m \times 0.833 g \sin q \le m_s (3.5 \mathrm{mg} \cos q + 0.41 \frac{h_1}{l} mg \sin q)$$

$$\tan q \le \frac{3.5 m_s}{0.5831 - 0.41 m_s \frac{h_1}{l}}$$

(17) 0.1 pts

CASE ALL CYLINDER SLIDING

From eq. (4) $f_{21h} = Ma + u_k N_2 - Mg\sin\theta$ (18) 0.15 pts

From eq. (8) $f_{31h} = Ma + u_k N_3 - Mg\sin\theta$ (19) 0.15 pts

From eq. (18) and 19 :

$5Mg \sin\theta - (Ma + u_k N_2 - Mg \sin\theta) - (Ma + u_k N_3 - Mg \sin\theta) = m_1 a$

$$a = \frac{7Mg \sin q - m_k N_2 - m_k N_3}{7M} = g \sin q - \frac{m_k (N_2 + N_3)}{7M} \quad (20) \quad 0.2 \text{ pts}$$

$$N_3 + N_2 = 7Mg \cos q$$

From the above two equations we get :

$$a = g \sin q - m_k g \cos q \quad 0.25 \text{ pts}$$

The Conditions for complete sliding: are the opposite of that of pure rolling

$$f_2 \rangle m_s N'_2 \quad \text{and} \quad f_3 \rangle m_s N'_3$$

$$\frac{I_2}{R_2^2} a \rangle m_s N'_2 \quad \text{and} \quad \frac{I_3}{R_3^2} a \rangle m_s N'_3 \quad (21) \quad 0.2 \text{ pts}$$

Where N_2' and N_3' is calculated in case all cylinder in pure rolling. 0.1 pts

Finally weget

$$\tan q \rangle \frac{3.5 m_s}{0.5831 + 0.41 m_s \frac{h_1}{l}} \quad \text{and} \quad \tan q \rangle \frac{3.5 m_s}{0.5831 - 0.41 m_s \frac{h_1}{l}} \quad 0.2 \text{ pts}$$

The left inequality finally become decisive.

CASE ONE CYLINDER IN PURE ROLLING AND ANOTHER IN SLIDING CONDITION

{ For example R_3 (front cylinders) pure rolling while R_2 (Rear cylinders) sliding}

From equation (4) we get

$F_{21h} = m_2a + u_kN_2 - m_2g \sin\theta$ (22) 0.15 pts

From equation (5) we get

$f_{31h} = m_3a + (I/R^2)a - m_3g \sin\theta$ (23) 0.15 pts

Then from eq. (1) , (22) and (23) we get

$m_1g \sin\theta - \{ m_2a + u_kN_2 - m_2g \sin\theta\} - \{m_3a + (I/R^2)a - m_3g \sin\theta\} = m_1a$

$m_1\ g \sin\theta + m_2g \sin\theta + m_3 \sin\theta - u_kN_2 = (I/R^2 + m_3)a + m_2\ a + m_1\ a$

$5Mg \sin\theta + Mg \sin\theta + Mg \sin\theta - u_kN_2 = (0.7M + M)a + Ma + 5Ma$

$$a = \frac{7Mg \sin q - m_k N_2}{7.7M} = 0.9091g \sin q - \frac{m_k N_2}{7.7M} \qquad (24) \qquad 0.2 \text{ pts}$$

$$N_3 - N_2 = \frac{h_1}{l}(m_k N_2 + \frac{I}{R^2}a + 2Ma - 2Mg \sin q)$$

$$N_3 - N_2 = \frac{h_1}{l}(m_k N_2 + 2.7M \times 0.9091g \sin q - 2.7m_k N_2 / 7.7 - 2Mg \sin q)$$

$$N_3 - N_2(1 + 0.65m_k \frac{h_1}{l}) = 0.4546Mg \sin q$$

$$N_3 + N_2 = 7Mg \cos\theta$$

Therefore we get

$$N_2 = \frac{7Mg \cos q - 0.4546Mg \sin q}{2 + 0.65m_k \frac{h_1}{l}}$$

$$N_3 = 7Mg \cos q - \frac{7Mg \cos q - 0.4546Mg \sin q}{2 + 0.65m_k \frac{h_1}{l}}$$

(25) 0.3 pts

Then we can substitute the results above into equation (16) to get the following result

$$a = 0.9091g \sin q - \frac{m_k N_2}{7.7M} = 0.9091g \sin q - \frac{m_k}{7.7}\frac{7g \cos q - 0.4546g \sin q}{2 + 0.65m_k \frac{h_1}{l}} \qquad (26)$$

0.2 pts

The Conditions for this partial sliding is:

$$f_2 \le m_s N_2' \qquad \text{and} \qquad f_3 \rangle m_s N_3'$$

$$\frac{\mathrm{I}}{\mathrm{R}^2}\mathrm{a} \le m_s N_2' \qquad \text{and} \qquad \frac{\mathrm{I}}{\mathrm{R}^2}\mathrm{a} \rangle m_s N_3' \qquad (27) \qquad 0.25 \text{ pts}$$

where N_2' and N_3' are normal forces for pure rolling condition

4. Assumed that after rolling d meter all cylinder start to sliding until reaching the end of incline road (total distant is s meter). Assummed that ηmeter is reached in t_1 second.

$$v_{t1} = v_o + at_1 = 0 + a_1 t_1 = a_1 t_1$$

$$d = v_o t_1 + \frac{1}{2} a_1 t_1^2 = \frac{1}{2} a_1 t_1^2$$

$$t_1 = \sqrt{\frac{2d}{a_1}}$$

0.5 pts

$$v_{t1} = a_1 \sqrt{\frac{2d}{a_1}} = \sqrt{2da_1} = \sqrt{2d0.833g \sin q} = \sqrt{1.666dg \sin q} \qquad (28)$$

The angular velocity after rolling d meters is same for front and rear cylinders:

$$w_{t1} = \frac{v_{t1}}{R} = \frac{1}{R}\sqrt{1.666dg \sin q} \qquad (29)$$

0.5 pts

Then the vehicle sliding untill the end of declining road. Assumed that the time needed by vehicle to move from d position to the end of the declining road is t_2 second.

$$v_{t2} = v_{t1} + a_2 t_2 = \sqrt{1.666dg \sin q} + a_2 t_2$$

$$s - d = v_{t1} t_2 + \frac{1}{2} a_2 t_2^2$$

$$t_2 = \frac{-v_{t1} + \sqrt{v_{t1}^2 + 2a_2(s-d)}}{a_2} \qquad (30) \qquad 0.4 \text{ pts}$$

$$v_{t2} = \sqrt{1.666dg \sin q} - v_{t1} + \sqrt{v_{t1}^2 + 2a_2(s-d)}$$

Inserting v_{t1} and a_2 from the previous results we get the final results.

For the angular velocity, while sliding they receive torsion:

$$t = m_k NR$$

$$a = \frac{t}{I} = \frac{m_k NR}{I} \qquad (31)$$

$$w_{t2} = w_{t1} + a t_2 = \frac{1}{R}\sqrt{1.666\, dg \sin q} + \frac{m_k NR}{I} \frac{-v_{t1} + \sqrt{v_{t1}^2 + 2a_2(s-d)}}{a_2}$$

0.6 pts